Robustness of the Maximal Covering Location Problem

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Emergency Medical Service

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The organisation and coordination of out-of-hospital:

- Acute medical care
- Transportation of patients

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Service providers are responsible for:

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Situation in The Netherlands:

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Services are tasked to optimise their performance

Placement of Ambulance Bases

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Use optimisation models to determine optimal base locations

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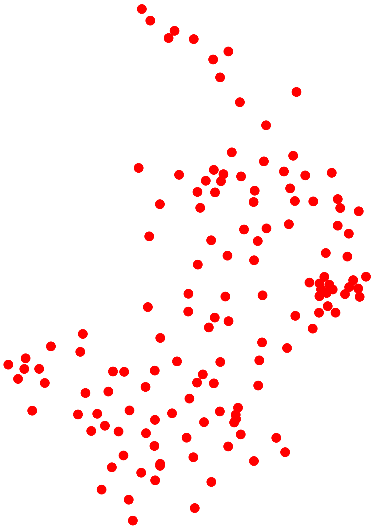

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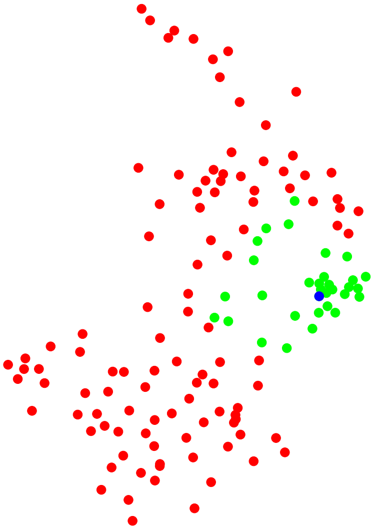

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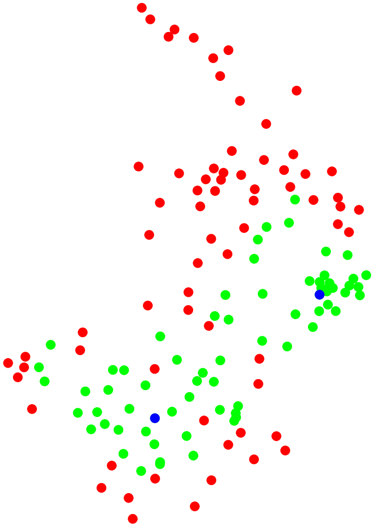

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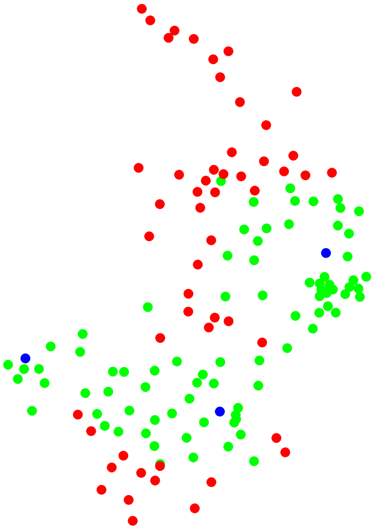

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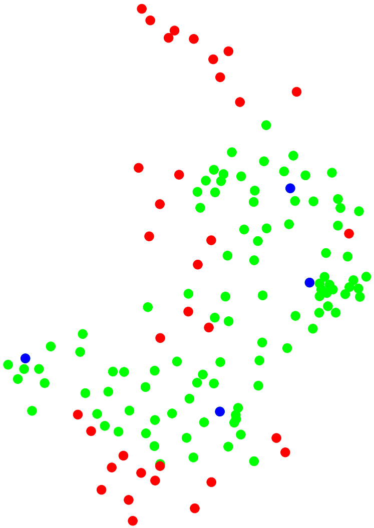

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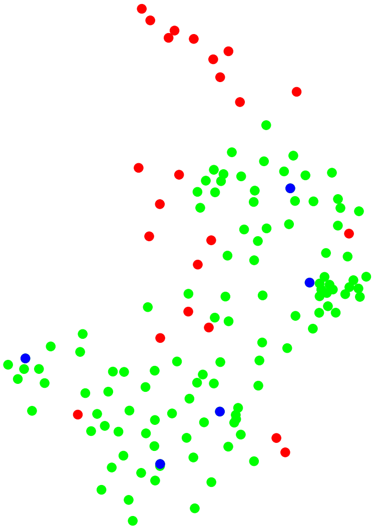

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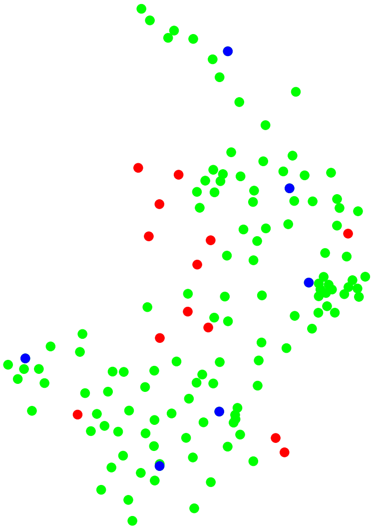

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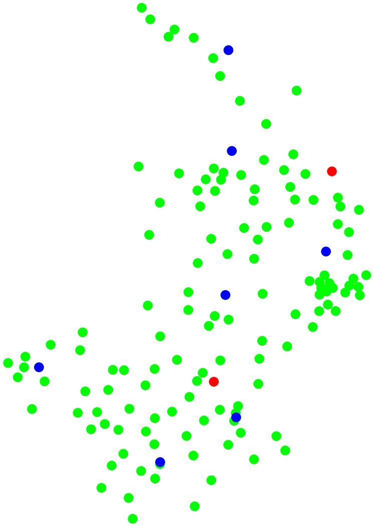

Placement of Ambulance Bases

Objective

$$\max \sum_{j \in \mathcal{J}} d_j z_j$$

Constraints

$$\sum_{i \in \mathcal{I}} x_i = p$$

$$\sum_{i \in \mathcal{I}} a_{ij} x_i \geq z_j \quad \forall j \in \mathcal{J}$$

$$x_i \in \mathbb{B} \quad \forall i \in \mathcal{I}$$

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Variables:

- Opened bases x_i
- Covered points z_j

Model Robustness

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Focus on demand weights d_j

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- Up to 5% deviation in demand d_j is possible
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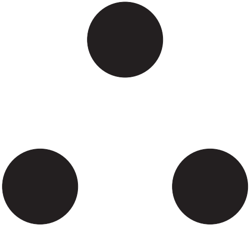

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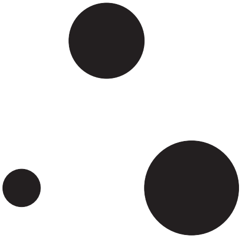

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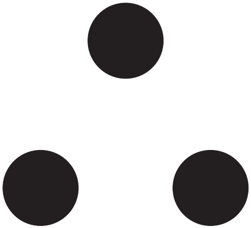

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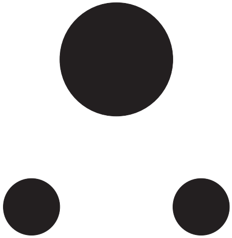

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Use Robust Optimisation techniques

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Use Robust Optimisation techniques

- Uses duality theory

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Suppose:

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Robust objective:

$$\underbrace{\Delta u}_{\text{estimated total coverage}} \underbrace{- \sum_{j \in \mathcal{J}} \overline{d}_j v_j^+}_{\text{correction overestimates}} \underbrace{+ \sum_{j \in \mathcal{J}} \underline{d}_j v_j^-}_{\text{correction underestimates}}$$

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Also some additional constraints

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We have two models:

- Normal coverage model
- Robust coverage model

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The model is robust with respect to demand uncertainty

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Alternative parameter uncertainty:

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Alternative parameter uncertainty:

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- Total demand is fixed
- Estimated demand $\hat{d}_j$ is the rate of Poisson process

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Same robust coverage model, but with:

$$d_j \in [\underline{d}_j, \overline{d}_j] = \left[\hat{d}_j - \sqrt{\hat{d}_j}, \hat{d}_j + \sqrt{\hat{d}_j}\right]$$

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Two models:

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Numerical results:

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Numerical results:

- Solution differs in 32 of 609 cases (5%)

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- Coverage difference is very small $(< 1\%)$

Conclusion

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Maximal Covering Location problem (MCLP):

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- Coverage optimisation model

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- Robust for two common types of demand uncertainty

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Robust Optimisation:

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- Useful for worst-case robust solutions
- See also:

Ben-Tal, A., L. El Ghaoui, and A. Nemirovski (2009). *Robust optimization*.

Questions?